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AEROSPACE SYSTEMS AND MISSION ANALYSIS RESEARCH

Quarterly Status Report for the Period

1 October through 31 December 1965

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AEROSPACE SYSTEMS AND MISSION ANALYSIS RESEARCH*

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I. INTRODUCTION

Work continued during the last quarter of 1965 with the same emphasis as previously reported. The heliocentric portions of the Earth-Mars and Mars-Earth trajectories have received intensive study with some accomplishments registered but with some shortcomings remaining before the Earth-Mars round trips can be computed.

Personnel with assignments in this Research Program are shown in APPENDIX A. Dr. Lion has undertaken to coordinate the work on spaceflight trajectory analysis now that Dr. Handelsman's participation has been reduced. A computer programmer and computer aide have been budgeted to accelerate the development of our trajectory programs and subsequently the mission analyses.

Student participation, graduate and undergraduate, has been stabilized during this period. Mr. G. A. Hazelrigg continues in his studies of planetary escape and capture as a preliminary to his doctoral thesis effort. Mr. S. M. Rocklin is undertaking an MSE thesis in the area of three-dimensional orbit transfers. Messrs. Peltier, Duffek and Crowley are in the early stage of their association. The undergraduate students are taking a significant part in the research. In addition to Mr. J. C. Slaybaugh and Mr. T. C. Hanks mentioned in the previous report, Mr. M. J. Flynn has initiated a study of nuclear rockets as discussed below and Messrs. T. R. Armstrong and D. R. Hansmann are working on the assemblage of programs for the round trip mission analyses, especially the interfaces.

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II. SPACEFLIGHT TRAJECTORY ANALYSIS RESEARCH*

Work continued through this period on the impulsive-iterative method for the determination of initial values of the Lagrangian multipliers. Dr. Handelsman is preparing a paper on the body of work accomplished to date and has been invited to present it at the AIAA Third Aerospace Sciences Meeting in New York City in January and has also offered it to the AIAA Journal. It will be issued as a report under this contract as listed in APPENDIX B.

The major emphasis on spaceflight trajectory analysis during the period was on the extension of the impulsive-iterative method to the multi-burn cases. At the beginning of the period the capability existed for computing two-dimensional, heliocentric transfers when the thrust program was on-off-on. The initial values of the adjoint vectors were estimated using the optimal; i.e., minimum propellant consumption, impulsive trajectory between the same terminals. This technique, at present, is limited as to the central angle which can be prescribed. For central angles of greater than π radians, the magnitude of the primer vector becomes positive during a finite portion of the two-impulse trajectory. This indicates that the two-impulse trajectory is no longer the optimum impulsive trajectory, and therefore, two-impulses will not be the limit point of optimum finite thrust trajectories with progressively lower thrust levels. It is conjectured then that a three-impulse trajectory may be the limit for certain optimum trajectories with central angles between π and 2π radians.

A program has been developed which determines the optimum three-impulse trajectory with fixed travel time, fixed central angle. It was, indeed,

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*Mr. William E. Miner, NASA Electronics Research Center, is the Technical Monitor of this phase of the research.

found that for certain ranges of central angle the total ΔV for the three-impulse trajectory was better than the corresponding two-impulse trajectory. Note that this does not imply that four (or even more) impulses might not be still better. However, the primer vector, calculated using Lawden's equations, satisfied all the necessary conditions - most importantly, the switching function was tangent to the zero line at the time of the middle thrusting - for the cases computed. It may be concluded then with some confidence (although not with absolute certainty) that these three-impulse trajectories are at least locally optimal. So far it has been impossible to converge on the finite thrust trajectories using the adjoint vectors of the three-impulse trajectories as starting iterates. This may be caused by one of two reasons:

1. The three-impulse trajectory is not a limit point in the sense described above.

2. The end point iteration scheme being utilized is not sufficiently "strong" to overcome the extreme sensitivity of trajectories with three thrusting periods to initial values of the adjoint.

Work will continue on the multi-burn trajectories and other extensions of the impulsive-iterative method, including three-dimensional transfers and angles around π for a wide variety of trip times, thrust-to-mass ratios, and jet velocities.

Computer programs for optimal Earth-Jupiter heliocentric transfers are being exercised and extended in their capabilities for a number of missions to the Jovian planet.

Research on planetary escape and capture trajectories continues for a wide range of various propulsion systems and combinations thereof. Besides adapting the existing methods described in MARS Memo #21, the formulation of the problem in multi-body terms for fully developed optimization analysis is being undertaken.

Work also continues on the effort to develop efficient and effective methods of hybrid computation for heliocentric transfers but no significant results can be reported at this time.

A series of seminars by leading authorities has been undertaken on a bi-weekly basis. This series has attracted considerable interest, not only within the trajectory analysis group, but throughout the Engineering School. A list of speakers and their topics for the Fall Term is attached as APPENDIX C.

III. AEROSPACE SYSTEMS ANALYSIS RESEARCH

Work on this research topic continued as previously outlined. Mr. M. J. Flynn, '67, has been assigned to the study on the sizing of conventional nuclear rocket systems.

Negotiations are in process for the presentation of a series of seminars on the use of computing machines for aerospace systems analysis, especially nuclear propulsion systems. Details will be presented in the next status report.

IV. PLANETARY/INTERPLANETARY MISSION ANALYSIS RESEARCH

While it is still not possible to assemble the trajectory programs that will permit analyzing the unmanned Earth-Mars round trip in adequate details, an effort is being made in the meantime to establish the overall program structure including optimization across the interfaces between the several sections. This work is being undertaken, with the help of the programming staff, by two senior undergraduates, Messrs. T. R. Armstrong and D. R. Hansmann for the initial mode of analysis of the unmanned Earth-Mars round trip.

Dr. Handelsman is now beginning preliminary studies that hopefully will lead to the definition of analytical methods for Jupiter missions.

APPENDIX A: Personnel Assignment List, As of 31 October 1965

Spaceflight Trajectory Analysis	Lion, P.M., Sloan Post-doctoral Fellow	pt,nc
	Wallack, P.J., Mathematician/Programmer	3/4t
	Hoffman, L., Programmer	1/4t
	( ), Programmer	ft
	( ), Computer Aide	ft
	Hazelrigg, G.A., Graduate Student (PhD Cand)	1/2t
	Rocklin, S.M., Graduate Student (MSE Cand)	1/2t
	Peltier, J.P., Graduate Student (MSE Cand)	1/2t
	(Duffek, W., NASA International Fellow	pt,nc)
	Consultants:	
	Crocco, L., Professor	
	Graham, D., Associate Professor	
	Bryson, A.E., Prof. (Harvard/MIT)	
	Leitmann, G., Prof. (U. of Cal., Berkeley)	
	RCA - Handelsman, M.	
	EAI - Vichnevetsky, R.	
	AMA - Pines, S., Kelley, H., etc.	
Propulsion System Analysis	Layton, J.P., Senior Research Engineer	3/8t
	Grey, J., Associate Professor	1/9t
	Jahn, R. G., Associate Professor	pt,nc
	Williams, P., Research Staff Member	pt
	(Crowley, F.B., Graduate Student (USAF)	pt,nc)
	Flynn, M.J., Undergraduate Student '67	pt,nc
	Slaybaugh, J.C., Undergraduate Student '66	pt,nc
Mission Analysis	Layton, J.P., Senior Research Engineer	3/8t
	Wallack, P.J., Mathematician/Programmer	1/4t
	Armstrong, T.R., Undergraduate Student '66	pt,nc
	Hanks, T.C., Undergrad. Student(Geol.Eng)'66	pt,nc
	Hansmann, D.R., Undergraduate Student '66	pt,nc
	Obi, W.C., Undergraduate Student '66	1/4t
Administrative	Allison, F., Senior Project Secretary	ft
	( ), Project Secretary	ft

APPENDIX B: List of Publications, As of 31 December 1965

1. Constantine, R. W., An Analysis of a Ramjet Propelled Recoverable Launch Vehicle Stage, (Senior Thesis - June 1964), Princeton Aeronautical Engineering Report No. 717a, (Limited Distribution), 3 June 1964.

2. Richardson, W. P., Investigations of a Hybrid Rocket Powered Unmanned Mars Excursion Vehicle, (Senior Thesis - June 1964), Princeton Aeronautical Engineering Report No. 717b, (Limited Distribution), 3 June 1964.

3. Handelsman, M., Hazelrigg, G. A. Hoffman, L. L. and Wallack, P. J., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 1, Overall Presentation, Princeton Aeronautical Engineering Report No. 717c-1, 29 January 1965.

4. Wallack, P. J., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 2, Computer Program, Princeton Aeronautical Engineering Report No. 717c-2, 29 January 1965.

5. Hoffman, L. L., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 3, Tabultated Results for Earth-to-Mars Transfer, Princeton Aeronautical Engineering Report No. 717c-3, 29 January 1965.

6. (Burton, C. D. and Evans, J. A., Nuclear Space Power System Analyses for the Unmanned Mars Round Trip, (Joint MSE Thesis - May 1965), Princeton Aeronautical Engineering Report No. 717d, (Limited Distribution), 20 May 1965).

7. Handelsman, M., Optimal Free-Space Fixed-Thrust Trajectories Using Impulsive Trajectories as Starting Iteratives, Princeton Aeronautical Engineering Report No. 717e. (In preparation).

APPENDIX C: List of Space Flight Trajectory Analysis Seminars, As of 28 October 1965

Date	Lecturer	Topic
11/3/65	Morris Handelsman, Radio Corporation of America	Impulsive-Iterative Approach to Orbital Transfers
11/15/65	George Leitmann, University of California - Berkeley	A Geometric Approach to Optimization
12/1/65	Paul Kenneth, Grumman Aircraft Engineering Corporation	The Newton-Raphson Operator Algorithm, and Some Related Thoughts on Functional-Analytic Approaches to Optimization Theory
12/17/65	Arthur E. Bryson, Harvard University	The Successive Sweep Method for Solving Optimal Programming Problems